Redundant Configuration of Electric Propulsion Systems for Stationkeeping

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Reliability of an electric propulsion system for north-south and east-west correction of a synchronous satellite depends on the redundancy which can be achieved with a given number of thrusters. Assuming a communication satellite with rotating solar panels in its north-south axis, four ion thrusters can be arranged inclined to this axis, giving north-south, east-west or radial thrust components. Several of such configurations are analyzed with respect to normal and redundant (after failure) operating modes, allowable thruster failure combinations and resulting reliability. This results in a choice of the best configuration and in reliability requirements for the single thruster.

I. Introduction

THE use of electric propulsion systems for stationkeeping of commercial synchronous satellites will depend on the reliability which can be demonstrated. The task of reliability development divides into two rather independent areas, namely 1) the practical development of a realiable ion thruster and its components, and 2) the design of a reliable stationkeeping system, making use of redundant operating modes of the thrusters to increase system reliability.

The second point is treated in this paper. If stationkeeping in the north-south and east-west directions (against the perturbations of sun and moon and the elliptic shape of the Earth's equator) is performed with separate thrusters, this can be achieved with a minimum of two thrusters, one (~ 5 mN) pointing in the north or south direction and the other ($\sim 100\ \mu$N) in the west direction or east direction, depending on the geographical longitude of the satellite position. If, as is usual, the initial positioning shall be performed, too, then in this system at least three thrusters are necessary to give the possibility of east and west thrust.

Such a system has the disadvantage that the failure of any single thruster leads to a failure of the system. The term failure of the system shall be defined by the condition that the system is not able to keep the satellite on station in the east-west and north-south directions within the position tolerance for a time only limited by the fuel supply.

The prime postulation in this paper is that the failure of any single thruster should not cause a failure of the system. This can only be achieved with a minimum number of four thrusters.‡ The redundancy which can be achieved with four thrusters depends on the thrust directions, i.e., the thruster positions at the satellite (configuration). With a given thruster configuration, a variety of operating modes are possible for north-south and east-west corrections using either all thrusters or a reduced number of thrusters after single or double thruster failures. Finding possible operating modes requires some orbit mechanics calculations. All possible operating modes of a certain thruster configuration determine the reliability of the system, i.e., the probability not to fail, calculated by probability theory. In addition, the capability of the thruster system of unloading a set of momentum wheels for attitude control by ion beam deflection is also considered. If two thruster configurations turn out to be equally redundant with respect to stationkeeping, the choice in this paper is made with respect to attitude control.

II. Configurations

The two basic types of future large synchronous communication satellites, the Earth-pointing satellite with rotating solar panels and the sun-pointing satellite with rotating antenna, have rather different aspects in arranging the stationkeeping thrusters. This paper concentrates on the first type. Figure 1 gives the principal design, resembling, e.g., the satellite projects ECS (European communication satellite ESRO) and CTS (Communication technology satellite, Canada–USA).

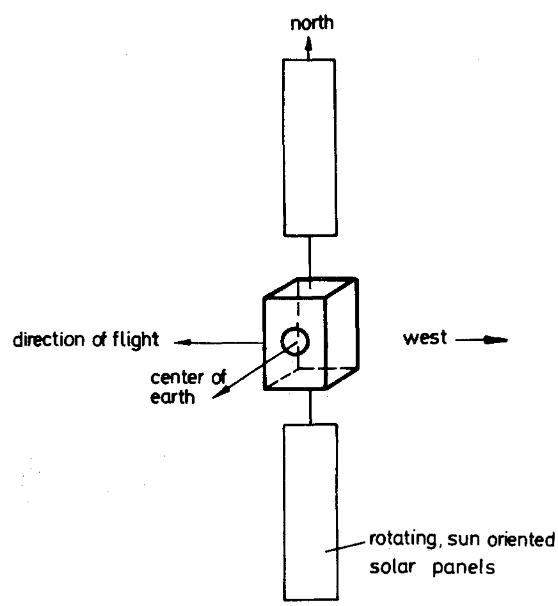

Fig. 1 Basic configuration of a synchronous communication satellite.

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‡ With this philosophy we follow a suggestion of W. A. Selke, ERNO-Raumfahrttechnik, Bremen, Germany, to whom we express our thanks for discussion.

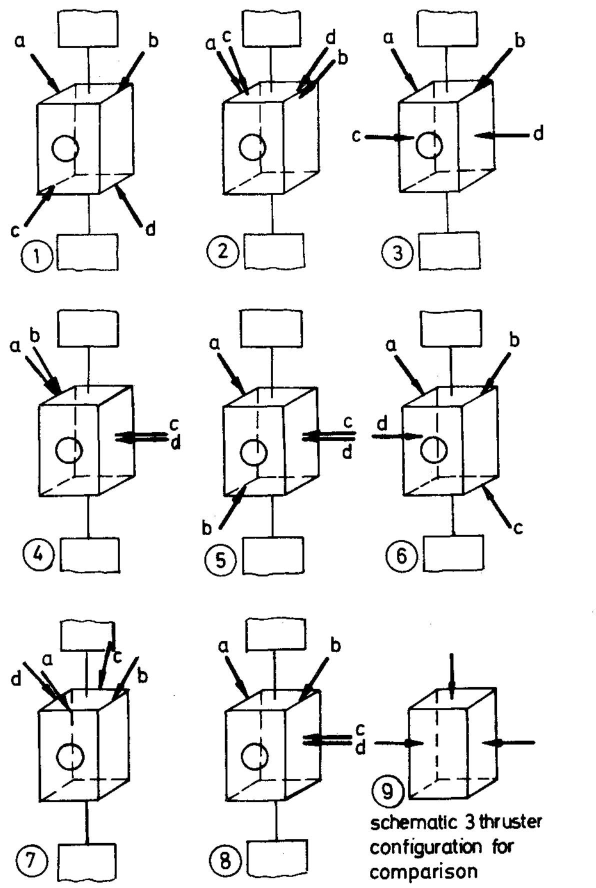

Fig. 2 Possibilities of arranging four ion thrusters for a stationkeeping system at a synchronous satellite. The arrowheads indicate the thruster positions (the arrows giving the thrust directions) in the undeflected condition all through the center of mass.

Other basic data for this paper are satellite mass = 800 kg; stationkeeping tolerance = 0.1° in north-south and east-west directions; stationkeeping velocity requirements are: $(\Delta v)_{NS} =$ 49 m/sec per year for north-south, $(\Delta v)_{EW} = 1.82$ m/sec per year for east-west, European longitudes. The north-south correction thrusters have to be allocated off the north-south axis of the satellite and must be canted by at least 30° against this axis because of interference with the solar panels.[1] Therefore, their thrust vector has either a radial or an east (or west) component, the latter suggesting their simultaneous use for east-west correction. At the same time this double use of the thruster increases redundancy, as will be seen. The whole system can be set up of four canted thrusters or of canted and pure east- and west-facing thrusters.

Figure 2 gives the possibilities under consideration. Many other possible configurations are excluded by the postulation that no single thruster failure shall lead to a system failure. This postulation also excludes configurations 7 and 8, since failure of thruster b in 7 and thruster a in 8 cause a system failure. Among configurations 1–6, the choice has to be made on the basis of a more detailed reliability analysis, discussed later in Sec. IV. The simple three-thruster configuration without redundancy discussed in Sec. I has been added as configuration 9 for comparison.

III. Correction Cycles

A. North-South Correction

As described elsewhere,[2,3] north-south correction is achieved by giving thrust vertical to the orbital plane in the vicinity of the nodes of the disturbed orbit, Fig. 3, either at one node only or with alternate thrust directions at both nodes. Such one-node-thrust can be generated, for instance, by simultaneous thrusting of thrusters a and b in configuration 1, Fig. 2, or of thrusters a and c in configuration 4. The configurations 1, 5, and 6 have also the capability of thrust at both nodes. During this simultaneous thruster operation the east-west thrust components cancel out if, in the cases of configurations 3–6 and 8, the thrust of the thrusters c and d is adjusted to the east (or west) component of the other thruster.

It is obvious that several redundant operation modes are possible. For instance, if in the case of configuration 1 the thruster a fails, then the north-south correction can be continued with one-node operation of thrusters c and d. After that event, the additional failure of thruster b has no influence. Also, whether or not the additional failure of thruster d rather than thruster b is tolerable (so that the diagonal-acting thrusters b and c are in operation) will be discussed in Sec. III-C. A third thruster failure, of course, renders the system unable to correct all perturbations because the remaining east or west components of a single intact thruster results in fast longitudinal drifting of the satellite.

Similarly, in the case of configuration 3, Fig. 2, one can change from one-node operation with thrusters a and b to one-node operation with thrusters a and d or b and c, whichever is still intact.

The daily operation time of the north-south correction thrusters will vary during the year because of the different positions of sun and moon. But the mean daily operation time (either at one node or accumulated at both nodes) follows from

$$t_{NS} = \frac{[(\Delta v)_{NS}/365]m_{Sat}\beta}{F_{NS}\sin\beta} \tag{1}$$

where β = angle of thrust period, Fig. 3; $F_{NS} = 2 \cdot F_{\text{thruster}} \cdot \cos 30°$; and $(\Delta v)_{NS} = 49$ m/sec per year. This time will be of influence to the reliability calculation in Sec. IV.

An inclination error of 0.1° is built up by perturbations in about 43 days. So, after a complete failure of the north-south correction, this time allows for the launch of a new satellite, if one can assume negligible inclination error during the continuous north-south correction.

B. Combined East-West and North-South Correction

1) Operation at two nodes

In the European longitudinal range, the east-west perturbation due to the elliptic shape of the Earth's equator acts on the satellite like a small continuous retarding thrust with the acceleration

$$a_{EW} = (dv/dt)_{EW} = -0.58 \cdot 10^{-7} \text{ m/sec}^2$$

For small east-west drifting movements of the satellite off its original position the retarding acceleration remains nearly constant, and we can treat the drifting movement like the

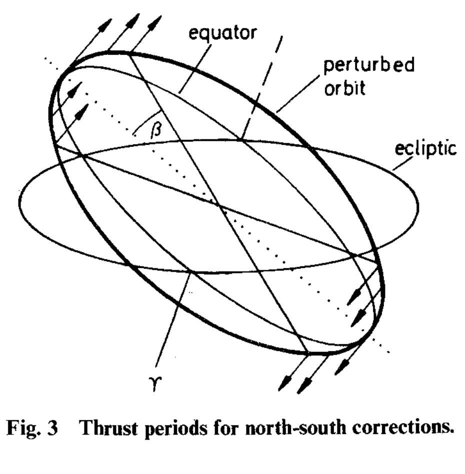

Fig. 3 Thrust periods for north-south corrections.

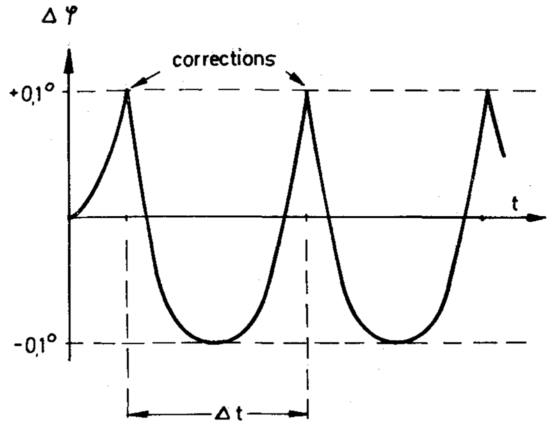

Fig. 4 East-west drift with corrections.

spiralling down with continuous thrust. Under the approximation that during this descent the instantaneous orbit is always circular, we obtain for the angular velocity after time t

$$\dot{\Psi}_c = v_c(t)/r_c(t) = (1/\mu)(v_{\text{synch}} + a_{\text{EW}} \cdot t)^3 \tag{2}$$

with v_c and r_c the velocity and the radius of the instantaneous orbit, respectively, and v_{synch} the circular velocity of the synchronous orbit. From that we obtain the angular displacement of the satellite relative to its original position

$$\Delta\Psi(t) = \int_0^t \dot{\Psi}_c(\tau)\, d\tau - \dot{\Psi}_{\text{synch}} \cdot t$$
$$= \frac{v^4_{\text{synch}}}{4\mu a_{\text{EW}}}\left[\left(1 + \frac{a_{\text{EW}}}{v_{\text{synch}}}t\right)^4 - 1\right] - \dot{\Psi}_{\text{synch}} \cdot t \tag{3}$$

where $\dot{\Psi}_{\text{synch}}$ is the angular velocity of the original and nominal position. Series expansion yields the results

$$t = [\tfrac{2}{3}\Delta\Psi \cdot (r_{\text{synch}}/a_{\text{EW}})]^{1/2} \tag{4}$$

for the time to drift through a given angular tolerance $\Delta\Psi$. Figure 4 shows the movement if corrections are performed at the border of the tolerance band. The longest possible time $(\Delta t)_{\max}$ between two corrections follows from Eq. (4) with $\Delta\Psi = 0.2°$:

$$(\Delta t)_{\max} = 30.1 \text{ days}$$

The east-west corrections have to be performed in these or shorter time intervals.

The east thrust duration is given by

$$t_{\text{EW}} = a_{\text{EW}} \cdot \Delta t \cdot m_{\text{sat}}/F_{\text{EW}} \tag{5}$$

where Δt = time between corrections, $F_{\text{EW}} = F_{\text{thruster}} \cdot \sin 30°$. The single correction has to be looked at as a small Hohmann transfer to lift the satellite to a higher orbit.

With the thruster configuration 1 in Fig. 2 it is very easy to perform such a Hohmann transfer in a daily rhythm. For this purpose, thruster b is started up somewhat earlier than thruster a during the daily north-south correction at the ascending node, and thruster d earlier than thruster c at the descending node. This gives two eastward thrust impulses at opposite points of the orbit. Figure 5 shows the thrusting periods. In this operating mode east-west drift can be nearly kept to zero at any time.

2) Operation at one node, eccentricity buildup

After a thruster failure in configuration 1 of Fig. 2, the north-south correction is continued at one node only, as mentioned before. In this case, and also in configurations 2–4, the operation mode of Fig. 5 is not possible.

If the east thrust is given at one node only by earlier startup of the respective thruster during north-south correction, then this does not represent a Hohmann transfer. Instead, eccentricity builds up, because each of the successive thrust periods is at the perigee of the transfer orbit. The eccentricity buildup follows the equation treated in Ref. 4

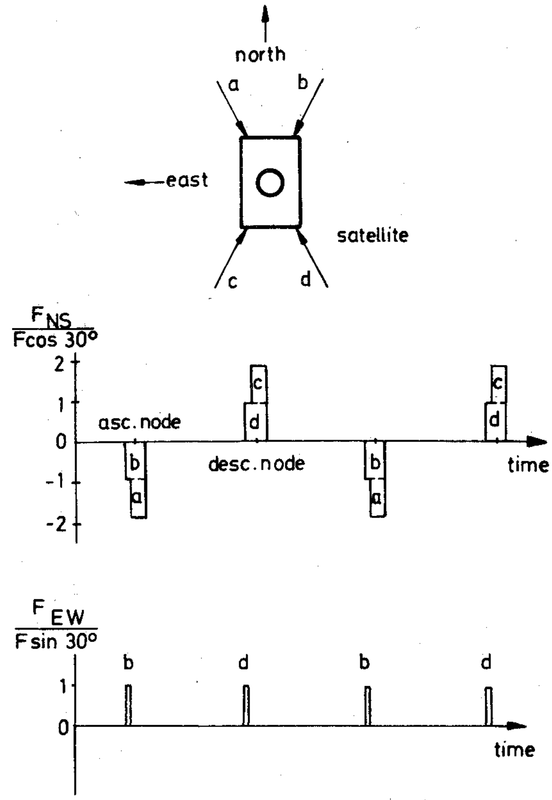

Fig. 5 Thrusting period for north-south (above) and east-west correction (below) with thruster configuration 1 in Fig. 2.

$$d\varepsilon/dv = (2/v)(\varepsilon + \cos\Psi) \tag{6}$$

where v is the velocity at the point of the orbit, where the tangential velocity increment dv is given, and where Ψ is the usual orbit angle, measured from the perigee.

Here we have $\Psi = 0$, $\varepsilon_0 = 0$, $v = v_{\text{synch}}$, so

$$\Delta\varepsilon = \varepsilon = (2/v_{\text{synch}})(\Delta v)_{\text{EW}} \tag{7}$$

The east-west-oscillation of the satellite due to this eccentricity can be derived from the difference in orbit angle between the actual eccentric satellite and the nominal point on the circular synchronous orbit, Fig. 6.

If we assume that at the moment of perigee passage P the actual satellite lies in the line of sight of the nominal point P', then the east-west oscillation is symmetrical around the nominal point, the largest angular displacement being between point 1

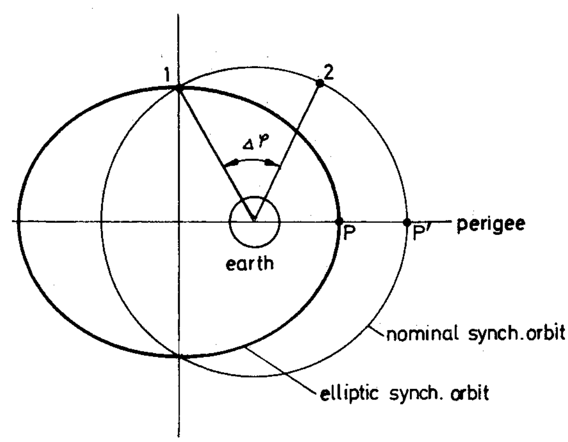

Fig. 6 For the calculation of maximum angular displacement $\Delta\Psi$ during east-west oscillation of an eccentric but synchronous satellite.

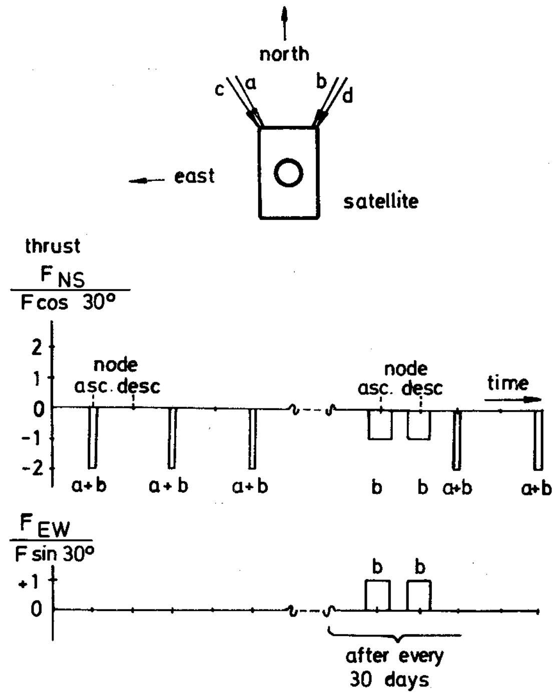

Fig. 7 Thrusting periods for north-south (above) and east-west correction (below) with the above thruster configuration (configuration 2 of Fig. 2).

and point 2 in Fig. 6. The time for travel through the orbit section $P \to 1$ is

$$t_{P\to 1} = \frac{r^3_{\text{synch}}}{\mu}\left[-\varepsilon\left(1-\left(\frac{v_{\text{synch}}-r_1}{r_{\text{synch}}\cdot\varepsilon}\right)^2\right)^{1/2} + \arc\cos\frac{r_{\text{synch}}-r_1}{r_{\text{synch}}\cdot\varepsilon}\right] \quad (8)$$

As here we have $r_1 = r_{\text{synch}}$, we obtain

$$t_{P\to 1} = (r^3_{\text{synch}}/\mu)[(\pi/2)-\varepsilon] \quad (9)$$

During this time the nominal point has travelled through the angle

$$\Psi_{P'\to 2} = (2\pi/T_{\text{rev}})\cdot t_{P\to 1} \quad (10)$$

which with Eq. (9) and

$$T_{\text{rev}} = 2\pi(r_{\text{synch}}/\mu)^{1/2}$$

yields

$$\Psi_{P'\to 2} = (\pi/2)-\varepsilon \quad (11)$$

As follows from the geometrics of Fig. 6, the actual satellite has travelled through the angle

$$\Psi_{P\to 2} = (\pi/2)+\varepsilon \quad (12)$$

if we remember that actually ε is small and so $\sin\varepsilon \approx \varepsilon$. Therefore the maximum angular displacement is

$$\Delta\Psi = 2\varepsilon \quad (13)$$

With Eq. (7) and $(\Delta v)_{\text{EW}} = a_{\text{EW}}\cdot t$ one obtains

$$t = (v_{\text{synch}}/4a_{\text{EW}})\,\Delta\Psi \quad (14)$$

for the time t, after which the east-west oscillation reaches an amplitude $\Delta\Psi$ if the east-west correction is done at one point of the orbit only (node of the north-south movement), for instance, with the thruster configuration 2 in Fig. 2. The angular tolerance of 0.1° would be reached by east-west oscillation after $t = 267$ days, even if the net drift were kept to zero.

Consequently one node thrusting is not possible for east-west correction for long periods. Therefore thrusting periods for configuration 2 in Fig. 2 (and similar) have to be chosen as in Fig. 7. After every 30 days of pure north-south correction, east-west correction is performed with thruster b at two successive nodes.

The south component of this thrust is "wrong" at the descending node; it cancels once every 30 days the corrective force at one node. This can be easily allowed for by slightly longer thrust periods during the rest of the 30 days.

C. Exclusion of Other Operating Modes

It is of interest whether or not other operating modes than those already discussed are practicable and might increase the redundancy after certain failures. In particular, there is the question of whether or not, in the configuration 1 in Fig. 2, the system is failed after thrusters a and d (or b and c) fail. North-south correction is still possible with the diagonally arranged thrusters b and c at both nodes, and one might believe that the east-west components of the thrust might cancel by alternate successive thrusting, giving the chance of east-west correction by varying the thrusting time ratio between the two thrusters.

This is not true, and it should be clear from the discussion in Sec. III-B, 2, that the successive accelerating and retarding thrust at opposite points of the orbit leads to fast eccentricity buildup with east-west oscillation following from it. Figure 8 (produced by analog simulation) shows the resulting satellite movement as seen from the Earth. So the operation modes are limited to those discussed in Sec. III-B, 2.

IV. Reliability

A. Choice of the Most Redundant Thruster Configuration

A comparison of reliability between the thruster configurations of Fig. 2 can be made without knowledge of the reliability values of the single thrusters. The discrimination is based on the number of redundant operating modes. A more detailed, time-dependent reliability analysis is done in Sec. IV-B. Here we simply assume the same reliability value R for each thruster.

In the thruster configurations 1–6, the system fulfills its task if either all four thrusters are intact, if any three out of the four thrusters are intact (configurations with system failure after one single thruster failure have been excluded), or if the system operates with two intact thrusters. Thus we get

$$R_{\text{syst}} = R^4 + (\tfrac{4}{3})R^3(1-R) + R_{\text{syst}}(2 \text{ failures}) \quad (15)$$

where R_{syst} (2 failures) is the probability that the system fulfills its duty with only two intact thrusters. System survival with only one intact thruster is impossible. The first two terms of Eq. (15) are the same for all the thruster configurations 1–6 (Fig. 2) so the

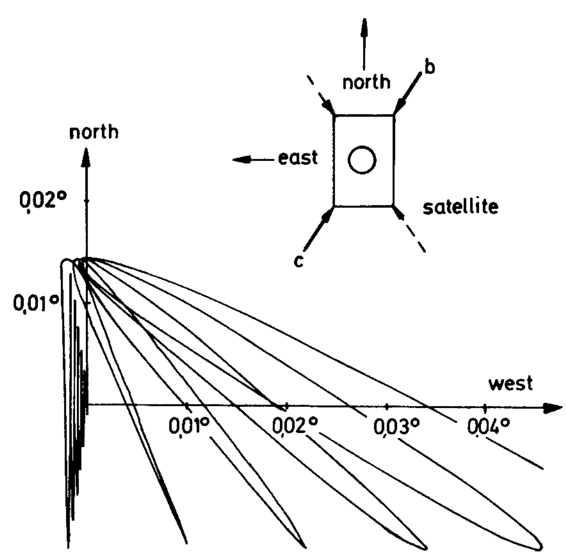

Fig. 8 East-west oscillation buildup caused by alternate operation of thrusters b and c at ascending and descending nodes of the disturbed orbit. North-south perturbation is corrected. The satellite movement (as seen from the Earth) starts with zero deviation at the origin of the coordinate system. Thruster operation begins after six days.

Table 1 The influence of dual thruster failures on the system capability of performing stationkeeping (+), momentum wheel discharge (+), or both (+**)**

configuration																																				
failure of thrusters	a+b	c+d	a+d	a+c	b+c	b+d	a+b	a+d	b+c	c+d	a+c	b+d	a+d	b+c	c+d	a+b	a+c	b+d	a+c	a+d	b+c	b+d	a+b	c+d	a+c	a+d	b+c	b+d	a+b	c+d	a+b	a+c	c+d	a+d	b+c	b+d
station keeping possible north south	+	+					+	+	+	+			+	+	+				+	+	+	+			+	+	+	+			+	+	+			
station keeping possible east west	+	+	+	+	+		+	+	+	+	+		+	+	+	+	+		+	+	+	+	+		+	+	+	+	+		+	+	+	+		+
momentum wheel discharge roll	+	+		+			+	+	+	+					+																		+	+		
momentum wheel discharge pitch	+	+		+			+	+	+	+			+	+	+	+	+		+	+	+	+	+		+	+	+	+	+		+	+	+	+		
momentum wheel discharge yaw	+	+	+	+	+		+	+	+	+	+		+	+	+	+	+		+	+	+	+	+	+	+	+	+	+	+		+	+	+	+		
system operable	**+**	**+**					**+**	**+**	**+**	**+**			+	+	**+**				+	+	+	+			+	+	+	+			+	+	**+**			

discrimination between the configurations is based on the last term. The operation modes for north-south and east-west stationkeeping with only two intact thrusters have been discussed in Sec. III-B. As a result, Table 1 gives the stationkeeping capabilities for all possible dual thruster failure combinations. The same compilation for the momentum wheel discharge capability by beam deflection is straightforward and is also in Table 1. The last line, "system operable," sums up the system survival cases, a boldface cross marking the capability of both stationkeeping and momentum wheel discharge. If n is the number of crosses in that line for a certain configuration, then the last term of Eq. (15) is

$$R_{\rm syst}\,(2\ \text{failures}) = (n/6)\cdot R^2(1-R)^2 \tag{16}$$

so the last term is directly proportional to the number of crosses. The highest number of crosses, namely four, is found in configurations 2, 4, and 5. But if momentum wheel discharge capability is included, configuration 2 is clearly superior, in this case configuration 1 is the second best.

The numerical difference in reliability values between these configurations should not be evaluated from the simple Eq. (15). This is solved in Sec. IV-B by a more detailed analysis.

B. System Reliability as a Function of Thruster Failure Rate

We assume the reliability (probability of survival) of the single thruster to obey the exponential law

$$R_{\rm on} = e^{-\lambda_{\rm on}t} \tag{17}$$

during operation and

$$R_{\rm off} = e^{-\lambda_{\rm off}t} \tag{18}$$

during switched-off condition, where $\lambda_{\rm on}$ and $\lambda_{\rm off}$ are the respective failures rates.

So after a mixed duty time of operation and waiting, the thruster reliability during duty is

$$R_d = e^{-\lambda_d t} \tag{19}$$

with

$$\lambda_d = (\lambda_{\rm on}\,t_{\rm on} + \lambda_{\rm off}\,t_{\rm off})/t$$

where $t_{\rm on}$, $t_{\rm off}$, and t are the operation, the waiting, and the total mission time. With the operation time ratio

$$k = t_{\rm on}/t$$

we can write

$$\lambda_d = \lambda_{\rm on}\,k + \lambda_{\rm off}(1-k) \tag{20}$$

We now analyze the best thruster configuration, configuration 2 of Fig. 2. It is assumed (without influence on the results) that at the beginning thrusters a and b are on duty (i.e., perform the correction cycles of Fig. 7) and thrusters c and d are on standby, i.e., in the off-condition. The system survives, if:

a) either "a" and "b" survive for the total time t;

b) or "a" fails during any time interval $t_1 \ldots t_1+dt_1$, and "c" is then intact and survives (now on duty) until t, and "b" survives for the total time t;

c) or "b" fails during any time interval $t_2 \ldots t_2+dt_2$ and "d" is then intact and survives (now on duty) until t, and "a" survives for the total time t;

d) or "a" fails during any time interval $t_1 \ldots t_1 dt_1$ and "c" is then intact and survives (now on duty) until t, and "b" fails during any time interval $t_2 \ldots t_2 dt_2$ and "d" is then intact and survives (now on duty) until t.

With the laws of probability theory, the reliability of the system at the end of the mission time t can now be expressed

$$R_{\rm syst} = [R_d(t)]^2 + 2R_d(t)\int_0^t R_{\rm off}(t_1)\cdot f_d(t_1)\cdot R_d(t-t_1)\,dt_1 + \left[\int_0^t R_{\rm off}(t_1) f_d(t_1) R_d(t-t_1)\,dt_1\right]^2 \tag{21}$$

with

$$f_d = -dR_d/dt = \lambda_d\,e^{-\lambda_d t} \tag{22}$$

With Eqs. (18, 19, and 22) one obtains

$$R_{\rm syst} = e^{-2\lambda_d t}[1+(\lambda_d/\lambda_{\rm off})(1-e^{-\lambda_{\rm off}t})]^2 \tag{23}$$

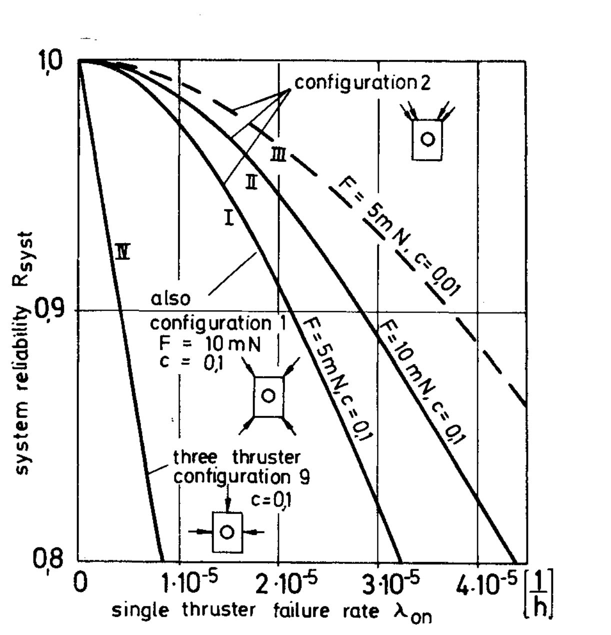

Fig. 9 The reliability of thruster systems for stationkeeping (and momentum wheel discharge): $\lambda_{\rm on}$ = single thruster failure rate during operation; $\lambda_{\rm off} = c\cdot\lambda_{\rm on}$ thruster failure rate during standby; F = thrust of single thruster.

Table 2 Thruster operation time ratios k

F mN	Configuration	k
5	1	0.0717
5	2	0.1435
10	1	0.0358
10	2	0.0717

A similar analysis is done for thruster configuration 1 of Fig. 2. For the numerical evaluation the mission time is $t = 7$ yr, the thruster failure rates are variables, keeping constant $\lambda_{\text{off}} = c \cdot \lambda_{\text{on}}$. From the correction cycles, the operation time ratios k defined in Eq. (20) are derived in Table 2.

Figure 9 shows the system reliability R_{syst} as function of the single thruster failure rate λ_{on}. The curves I, II, and III for the best four-thruster configurations 1 and 2 show the very large reliability increase against the simple three thruster configuration, curve IV, used for comparison. The reliability advantage of configuration 2 compared to configuration 1 can be seen from curves II and I. This advantage increases with higher thruster failures rates. At the same time, curves I and II show the difference in reliability requirements for 10 mN and 5 mN thrusters in configuration 2. If one postulates a reliability figure for the whole thruster system of $R_{\text{syst}} = 0.95$, then one would have to demonstrate a single thruster failure rate of

$$\lambda_{\text{on}} = 1.45 \cdot 10^{-5}/h$$

for 5 mN thrusters and

$$\lambda_{\text{on}} = 1.95 \cdot 10^{-5}/h$$

for 10 mN thrusters.

The dashed curve for $\lambda_{\text{off}} = 0.01 \cdot \lambda_{\text{on}}$ shows that the standby times and the off times during duty have considerable influence, which might further reduce the reliability requirements for the single thruster.

V. Conclusions

Among all possible four-thruster configurations, with configuration 2 of Fig. 2, the highest possible redundancy in performing stationkeeping and momentum wheel discharge is obtained. This leads to moderate reliability requirements for the single thruster, which are considerably lower than for a system without redundancy.

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